

RESEARCH SUMMARY

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My background is in systems control engineering and I am interested in optimization, control and analysis of dynamical systems, robotics, machine learning, and data processing. This statement summarizes my current and previous research work.

1. Current Research. During my PhD, my work has mainly focused on semidefinite programming and polynomial optimization and their applications to control and analysis of dynamical systems.

1.1. Chance Constrained Optimization. In this work, we address the chance optimization problems, where one aims at maximizing the probability of a set defined by polynomial inequalities [1, 2]. More precisely, we focus on the problem as the form of $\mathbf{P}^* := \sup_{x \in \mathbb{R}^n} \text{Prob}_{\bar{\mu}_q} \left(\bigcup_{k=1}^N \bigcap_{j=1}^{\ell_k} \{q \in \mathbb{R}^m : \mathcal{P}_j^{(k)}(x, q) \geq 0\} \right)$ where $q \in \mathbb{R}^m$ is random variable with given probability measure $\bar{\mu}_q$, $x \in \mathbb{R}^n$ is the decision variable, and $\mathcal{P}_j^{(k)} : \mathbb{R}^n \times \mathbb{R}^m \rightarrow \mathbb{R}$, $j = 1, 2, \dots, \ell_k$ and $k = 1, \dots, N$ are given polynomials. This class of problems encompasses many applications from different areas. For example, designing probabilistic robust controllers and model predictive controllers in presence of random disturbances, and optimal path planning and obstacle avoidance problems in robotics can be cast as special cases of this framework. Moreover, problems in the area of economics, finance, and trust design can also be formulated as chance optimization problems. Chance optimization problems are, in general, nonconvex and computationally hard. To solve such problems, we proposed a sequence of semidefinite programs of increasing dimension and showed that the sequence of optimal values converge to the optimal value of the original problem. In the proposed method, one needs to search for the positive Borel measure with maximum possible mass on the given semialgebraic set, while simultaneously searching for an upper bound probability measure over a simple set containing the semialgebraic set and restricting the Borel measure. Using theory of moments as well as duality theory, we provide a sequence of finite dimensional SDPs on moments and polynomials. To improve the numerical performance, an orthogonal basis of polynomials is employed, and to be able to efficiently solve the resulting large-scale semidefinite relaxations, a first-order augmented Lagrangian algorithm is implemented [1].

1.2. Probabilistic Robust Control. In this work, we address the problem of designing probabilistic robust controllers for discrete-time systems whose objective is to reach and remain in a given target set with high probability ([3], [1] see example 4 in section 5.3.5). More precisely, given probability distributions for the initial state, uncertain parameters and disturbances, we develop algorithms to design a control law that i) maximizes the probability of reaching the target set in N steps, and ii) makes the target set robustly positively invariant. Probabilistic robust formulation can be applied to different areas to deal with uncertainty and to ensure that the probability of failure/success is minimized/maximized, e.g., minimizing probability of obstacle collision in motion planning of robotic systems under environment uncertainty, minimization risk in the area of economics and finance. To solve this problem, a sequence of convex relaxations is provided which can arbitrarily approximate the solution of the probabilistic robust control design problem. We address this problem in two steps. To design a nonlinear state feedback control law, we first determine a set of parameters of control laws that renders the given target set robustly positively invariant. We approximate this set by a semialgebraic set where results in semidefinite programming involving sum of squares (SOS) polynomials. Then, we search for a control law in this set that maximizes the probability of reaching the target set in N steps. This step can be formulated as a chance optimization problem which also results in a semidefinite programming problem. While we maximize the probability of reaching the target set, simultaneously, we can maximize the probability of the different control objectives like probability of avoiding the obstacles, ([4], see section IV.C and example 2 in VII.B). We are currently extending this method to develop probabilistic robust model predictive controllers, where we aim at maximizing the probability of a predefined objective function over finite horizon under some probabilistic constrains [5].

1.3. Constrained Semialgebraic Volume Optimization: Application in System and Control. The purpose of this work is to develop a unified framework to generate LMI relaxations for different

problems in the area of system and control that are known to be nonconvex. For this, we introduce *constrained volume optimization* problems and show that many problems can be cast as a particular type of this problem [4]. In constrained volume optimization problem, we aim at maximizing the volume of a semialgebraic set under some semialgebraic constraints. More precisely, let $\mathcal{S}_1(a)$ and $\mathcal{S}_2(a)$ be semi-algebraic sets described by set of polynomial inequalities as $\mathcal{S}_1(a) := \{x : \mathcal{P}_{1j}(x, a) \geq 0, j = 1, \dots, o_1\}$ and $\mathcal{S}_2(a) := \{x : \mathcal{P}_{2j}(x, a) \geq 0, j = 1, \dots, o_2\}$ where a denotes the vector of design parameters, and let μ_x be a given finite nonnegative Borel measure. The objective is to find the parameter vector a such that the volume of the set $\mathcal{S}_1(a)$ becomes maximum while it is contained in the set $\mathcal{S}_2(a)$, i.e., $\mathbf{P}_{\text{vol}}^* := \sup_{a \in \mathcal{A}} \text{vol}_{\mu_x} \mathcal{S}_1(a)$, s.t. $\mathcal{S}_1(a) \subseteq \mathcal{S}_2(a)$. Many well-known problems from different fields can be formulated as a special case of this problem, and in this work, we focus on solving challenging problems in the area of system and control. Particularly, we address the problems of inner approximation of region of attraction (ROA) and maximal invariant sets of polynomial systems. We also introduce a new class of sum of squares (SOS) problems which enable us to find a strictly positive polynomial on some unknown semialgebraic sets. To solve the constrained volume optimization problem, a sequence of finite semidefinite programmings is provided, whose sequence of optimal values is shown to converge to the optimal value of the original problem [4].

1.3.1. Region Of Attraction Set. For a given polynomial system $\dot{x} = f(x)$, maximal region of attraction (ROA) set is the largest set of all initial states whose trajectories converge to the origin. This set can be approximated by level sets of Lyapunov polynomial function $V(x)$. The level set of Lyapunov function $\{x \in \mathbb{R}^n : 0 \leq V(x) \leq 1\}$ is ROA set if it is contained in the region described by $\{x \in \mathbb{R}^n : \dot{V}(x) \leq -\epsilon \|x\|_2^2\}$. By characterizing $V(x)$ with a finite order polynomial with unknown coefficients vector a and defining $\mathcal{S}_1(a) := \{x \in \mathbb{R}^n : 0 \leq V(x, a) \leq 1\}$ and $\mathcal{S}_2(a) := \left\{x \in \mathbb{R}^n : \dot{V}(x, a) \leq -\epsilon \|x\|_2^2\right\}$, the problem of finding maximal ROA set can be reformulated as a constrained volume optimization problem where μ_x is the Lebesgue measure of a simple set containing the sets [4]. The unknown vector a can also contains the parameters of the control law as well. With the same reasoning, one can extend the ROA problem for discrete time systems $x_{k+1} = f(x_k)$ by replacing the derivative of Lyapunov function $\dot{V}(x)$ with the difference Lyapunov function $\Delta V(x) = V(x_{k+1}) - V(x_k)$ [4].

1.3.2. Maximal Invariant Set. For a given polynomial system $x_{k+1} = f(x_k)$, where $x \in \chi_{ext}$ and given compact set $\chi \subset \chi_{ext}$, the invariant set in χ is the set of all initial states whose trajectories remains inside the set. This set can be approximated by the semialgebraic set $\mathcal{V} = \{x \in \chi : \mathcal{P}(x) \geq 0\}$, for a finite order polynomial $\mathcal{P}(x)$. The set \mathcal{V} is an invariant set for the dynamical system above if $\mathcal{P}(f(x)) \geq 0$ for all $x \in \mathcal{V}$. By characterizing $\mathcal{P}(x)$ with a finite order polynomial with unknown coefficients vector a and defining $\mathcal{S}_1(a) = \{x \in \chi : \mathcal{P}(x, a) \geq 0\}$ and $\mathcal{S}_2(a) = \{x \in \chi : \mathcal{P}(f(x), a) \geq 0\}$, the problem of finding maximal invariant set can be restated as a constrained volume optimization problem where μ_x is the Lebesgue measure of a simple set containing the sets [4].

1.3.3. Generalized Sum Of Squares. Using sum of squares (SOS) polynomial representation, we can find a polynomial that is strictly positive on the given semialgebraic set. In this work, we generalize the concept of SOS and introduce a new class of SOS problems which enable us to find a strictly positive polynomial on some unknown semialgebraic sets [4]. More precisely, we define generalized (SOS) problems as follow. Consider polynomial $\mathcal{P}(x, a)$ and semialgebraic set $\mathcal{S}_1(a) := \{x \in \chi : \mathcal{P}_{1j}(x, a) \geq 0, j = 1, \dots, o_1\}$ where $a \in \mathcal{A} \subset \mathbb{R}^m$ denotes the vector of unknown parameters. We aim at finding the parameter vector a such that polynomial $\mathcal{P}(x, a)$ is strictly positive on the set $\mathcal{S}_1(a)$. This problem can be restated as a constrained volume optimization problem by defining the set $\mathcal{S}_2(a) := \{x \in \chi : \mathcal{P}(x, a) \geq 0\}$ [4].

1.4. Reconstruction of Support of a Measure From Its Moments. In this work, we reconstruct the support of a measure from its moments [6]. More precisely, given a finite subset of the moments of a measure, we develop a semidefinite program to approximate the support using level sets of polynomials $\mathcal{K} = \{x \in \mathbb{R}^n : \mathcal{P}(x) \geq 1\}$. This problem has diverse applications in many different areas. A few examples include shape reconstruction from indirect measurements, signal reconstruction from sparse measurements, and problems in statistics. Moreover, this problem can be applied in the area of optimization. For example, in moment approaches to polynomial optimization, one aims at solving given optimization problem by looking at moments of the measures of equivalent convex problem. To extract an optimal solution of the

original problem, one needs to find a point in the support set of the optimal solution of the problem on measures. To solve support reconstruction problem, a sequence of convex relaxations is provided, whose optimal solution is shown to converge to the support of measure of interest. The proposed method relies on results on SOS polynomials and also, results on necessary and sufficient condition for moment sequence to have a representing measure. A modification of our approach aimed specifically at uniform distributions dramatically improves the obtained results [6].

1.5. Data Reconstruction. In this work, we address the problem of reconstruction of noisy and sparse data [7]. Given noisy n -dimensional sensor data, the objective is to denoise the data and estimate the missing points where the sensor data is not available. This problem arises in different areas like sensor networks, where the sensors do not completely cover the area of interest; hence, the sampled data are usually inadequate. Moreover, reconstruction of corrupted images and videos can be cast as special cases of this frame work. In this work, the objective is to find a convex formulation to complete the data with least possible complexity and take the topology information of the sensors in to account, as well. The complexity is defined as the order of linear difference equations describing the data or equivalently the number of exponential signals that could describe the data. From system theory perspective, we interpret the given n -dimensional data as a sequence of impulse response of a n -dimensional linear system. In this case, based on the theory of minimum system realization, the rank of block Hankel matrix constructed from the sparse data corresponds to the underlying system order. Hence, the problem of data reconstruction is reformulated as the problem of minimum rank block Hankel matrix completion problem, where the nuclear norm is used as a convex relaxation of the matrix rank. To be able to deal with large scale data, a first-order augmented Lagrangian algorithm is implemented for solving the resulting nuclear norm minimization problem. From signal theory perspective, we assume that the given n -dimensional data can be expressed as a weighted sum of complex exponential signals in n -dimensional frequency space. In this case, it can be shown that the rank of block Hankel matrix constructed by the data is equal to the number of exponentials describing the data. Hence, the problem of data reconstruction can be reformulated as the problem of minimizing the number of exponentials. To solve this problem, we use parsimonious model identification via atomic norm minimization concept. In this context the atoms are defined as exponential signals and then minimizing the atomic norm leads to sparse representations. To solve the resulted atomic norm minimization, randomized Frank-Wolfe algorithm is implemented. At each step, the algorithm requires computing only inner products and eliminates the need to singular value decompositions and thus computational time scales linearly with the size of the data. This enables us to deal with large scale data sets. Interpreting the given n -dimensional data as an output of a n -dimensional linear system and equivalently expressing it as a weighted sum of complex exponentials in n -dimensional frequency space not only results in convex optimization problem, but also enables us to consider the topology information of the sensors, as well.

2. Past Research.

2.1. Nonlinear Model Predictive Control for Manipulator Robots. In my master's thesis, the problem of path tracking and obstacle avoidance for redundant manipulator robots using nonlinear model predictive control (NMPC) is addressed [8, 9, 10, 11, 12]. Using the NMPC, the input voltages of DC servomotors of joints are obtained in such a way that the end-effector of a redundant manipulator tracks a given path in the Cartesian space considering obstacles and singularity avoidance. Nonlinear dynamic of the robot, including actuators dynamic, is also considered. At every sampling time k , based on measurements obtained at time k , the controller predicts the future output of the system over prediction horizon N_p using model of the system and determines the input over the control horizon $N_c \geq N_p$, such that a predefined cost function is minimized and defined constrains are satisfied. The defined cost function contains path tracking and obstacle avoidance terms and constrains involves singularity avoidance and limiting input voltage terms. Also, to improve the performance of the control, the online tuning of the weights in the NMPC is performed using **fuzzy logic**. The proposed method automatically adjusts the weights in the cost function in order to obtain good performance. The proposed fuzzy system uses distance between the manipulator and the obstacle and also the rate of change of this distance as the inputs. The outputs of the fuzzy system are the weights related to path tracking and obstacle avoidance in the cost function, [9, 11]. In the next step, to avoid collision with moving obstacles, the future position of the obstacles in 3D space is predicted using **artificial**

neural network. Using online neural network, no knowledge about obstacles motion is needed. Moreover, the end-effector of robotic manipulator, using this method, can capture a moving target, [10, 8]. Furthermore, to obtain a robust controller, an artificial neural network is implemented. A Multilayer Perceptron (MLP) is used for modeling nonlinear dynamic of the robot in MPC. Using neural network for model prediction, no knowledge about system parameters is needed and system robustness against change in parameters is obtained, [12]. Also, in this work the global **stability of nonlinear adaptive neural-MPC** is established. For this, a Lyapunov function based on control goals and identification error of neural networks is defined. Then, the adaptation laws for weights of the neural network in the prediction model are obtained in such a way that the stability of closed loop systems is guaranteed based on Lyapunovs direct method, [8].

2.2. Path Planning and Tracking for mobile and manipulator robots. In this project, different methods of path planning and tracking of mobile and manipulator robots are analyzed and compared. Using path planning algorithms, a feasible path between start and goal points is determined in such a way that no collision between robot and obstacles occurs. For manipulator robots, feasible paths for joints of manipulator are determined in such a way that end-effector of robot tracks predefined path or moves to goal point in the Cartesian space while at the same time avoids collision with obstacles and singular configurations. Some of studied algorithms for path planning are as follow: Roadmap, Cell Decomposition, Randomized Road Map, Rapidly Exploring Random Tree, Artificial Potential Fields, Optimization Based Algorithms, Task-priority Redundancy Resolution, Gradient Projection, Generalized Inverse Jacobin, Extended Jacobin approache. To track the obtained paths, different controllers like sliding mode controller as a robust controller, nonlinear model reference controller as an adaptive controller, and fuzzy controllers are designed. In this regard, I was also member of robosoccer team, during my undergraduate studies, that was responsible to build small-size **soccer robots**.

2.3. Fuzzy Controller Design Using Evolutionary and Swarm Algorithms. In my bachelor's thesis, the problem of optimal design of fuzzy controllers is addressed [13, 14]. In this problem, we aim at finding parameters of fuzzy controller such as fuzzy rule bases and membership function of the input and output variables such that the predefined control objectives are satisfied. For this, the controller design problem is reformulate as an optimization problem and then different evolutionary and swarm optimization algorithms are used. Particularly, Genetic Algorithm (GA), Cooperative Genetic Algorithm, Particle Swarm Optimization (PSO), and Imperialist Competitive Algorithm (ICA) are employed.

2.4. Intelligent Electronic Circuits Design. The purpose of this project is to implement artificial neural network control systems and evolutionary optimization algorithms to design electronic circuits. For example in [15, 16], neural multi-input multi-output (MIMO) controller and also genetic based linear controller are developed for an input-series and output-parallel (ISOP) Dc-Dc converter. The purpose is to improve the large signal performance of the ISOP converter in the case of large variations in the parameters or considerable dissimilarity between modules of converter. The proposed MIMO controller and linear controller are trained using particle swarm and genetic algorithms where eliminates the need for a prior knowledge of the system dynamics. Also, in [17], genetic algorithm is used to design of broadband microwave amplifiers. For this, compensated matching method is formulated as an optimization problem. In this case using GA, the parameters of input and output of matching networks are determined such that desired gain and band width are achieved.

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